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ENHANCING STUDENTS' GEOMETRIC THINKING THROUGH PHASE-BASED INSTRUCTION USING GEOMETER'S SKETCHPAD: A CASE STUDY

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Abstract: I investigated Form One students' learning of solid geometry in a phase-based instructional environment using Geometer's Sketchpad (GSP) based on the van Hiele theory. Specifically, I examined the students' initial van Hiele levels of geometric thinking about cubes and cuboids, and how their van Hiele levels changed after phase-based instruction with GSP. I used a case study research design and purposeful sampling to select six case study participants from a class of mixed-ability Form One students. Findings reveal that the participants' initial van Hiele levels ranged from Level 0 to Level 2. After phase-based instruction with GSP, their van Hiele levels either increased or remained the same.

Keywords: geometric thinking, phase-based instruction, Geometer's Sketchpad, van Hiele theory

Abstrak: Kajian ini bertujuan mengkaji pembelajaran geometri pepejal dalam kalangan pelajar Tingkatan Satu dalam persekitaran pengajaran berasaskan fasa dengan menggunakan *Geometer's Sketchpad* (GSP) berdasarkan teori van Hiele. Khususnya, kajian ini mengkaji tahap pemikiran geometri van Hiele awal pelajar tentang kubus dan kuboid, dan bagaimana tahap van Hiele pelajar berubah selepas pengajaran berasaskan fasa dengan menggunakan GSP. Pengkaji menggunakan reka bentuk kajian kes dan persampelan bertujuan untuk memilih enam peserta kajian kes daripada sebuah kelas Tingkatan Satu yang mempunyai pelajar-pelajar berlainan kebolehan. Dapatan kajian menunjukkan bahawa tahap van Hiele awal peserta berbeza-beza antara Tahap 0 dan Tahap 2. Selepas pengajaran berasaskan fasa dengan menggunakan GSP, tahap van Hiele peserta meningkat atau berada pada tahap yang sama.

Kata Kunci: pemikiran geometri, pengajaran berasaskan fasa, *Geometer's Sketchpad*, teori van Hiele

INTRODUCTION

The study of solid geometry is important for several reasons. First, solid geometry is a foundation for study in such fields as science, engineering, architecture, computer science, graphics, geology and astronomy (Banchoff, 1990; Senechal, 1990). Second, it provides a rich source of visualisation for understanding basic mathematical concepts (Sherard, 1981). Third, solid geometry improves students' perception of spatial relationships. Fourth, it provides continued growth and power in logical reasoning (Smith & Ulrich, 1957). Finally, there are cultural and aesthetic values to be derived from the study of solid geometry (O'Daffer & Clemens, 1992) that help students "appreciate the importance and beauty of mathematics" (Ministry of Education, 2003: 2).

Despite its importance, secondary students still performed poorly on the compulsory solid geometry questions in Mathematics Paper 2 of the Penilaian Menengah Rendah (PMR) and Sijil Pelajaran Malaysia (SPM) examinations (Malaysian Examinations Syndicate, 2004a, 2004b). In addition, Form Two students performed poorly on the geometry items at the Top 10% International Benchmark in TIMSS 1999 and at the High International Benchmark in TIMSS 2003. Their performance on the items was ranked 22 out of 38 participating countries in TIMSS 1999 (Mullis et al., 2000) and 19th out of 49 participating countries in TIMSS 2003 (Mullis et al., 2004). The rankings in TIMSS 1999 and TIMSS 2003 reflected Malaysian students' lack of geometric thinking ability. To address this concern, it is important to provide beginning secondary-school students with a strong foundation in solid geometry (Tay, 2003; van Hiele-Geldof, 1959/1984). Thus, I selected Form One students and the solid geometry chapter on cubes and cuboids for this study.

Importance of GSP

The Ministry of Education advocates the use of GSP in the teaching and learning of geometry (Ministry of Education, 2003) as it "can best foster mathematical inquiry and learning through 'dynamic manipulation' experiments" (Finzer & Jackiw, 1998: 2). GSP, with its dynamic manipulation environments, has three important attributes. First, students can directly manipulate mathematical objects represented on the screen. For example, students point at a cube vertex and can directly drag it from point *A* to point *B* (see Figure 1). Second, mathematical objects stay coherent at all times as they are dragged. Continuing the cube example, as the cube's vertex moves from point *A* to point *B,* students can see that the length of the edges and the orientation of the cube change continuously but the resulting figure will always be a cube. Third, students feel that they are involved with the objects they are manipulating: that is, they are immersed in the environment.

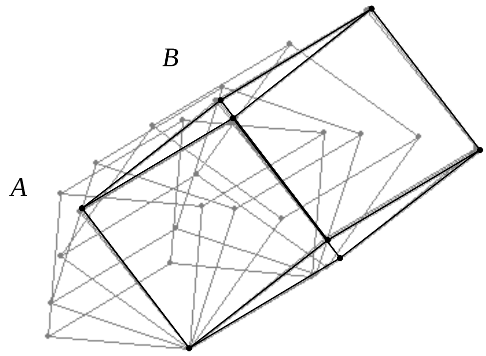

Figure 1. Dragging a cube in GSP

PURPOSE OF THE STUDY

The purpose of this study was to investigate Form One students' learning of solid geometry in a phase-based instructional environment using GSP based on the van Hiele theory. Specifically, this study aimed to answer the following research questions:

1. What were students' initial van Hiele levels of geometric thinking about cubes and cuboids?

2. How did students' van Hiele levels of geometric thinking about cubes and cuboids change after phase-based instruction with GSP?

THEORETICAL FRAMEWORK

The van Hiele Theory of Geometric Thinking comprises three main components; levels of geometric thinking, characteristics of the levels and phases of learning (Crowley, 1987).

Levels of Geometric Thinking

According to the theory of Pierre and Dina van Hiele (van Hiele, 1959/1984), students progress sequentially through five levels of thinking while learning geometry. This study limits its scope to the first three levels, for two main reasons. First, an analysis of the revised Form One mathematics syllabus and Form One mathematics textbooks showed that the content of the solid geometry chapter is only up to Level 2. Second, previous studies have shown that lower secondary students (aged 13–14 years old) are highly unlikely to attain Level 4 or 5 in solid geometry (Gutiérrez, Jaime & Fortuny 1991; Lawrie, Pegg & Gutierrez, 2000) and in plane geometry (Noraini 1998; Tay, 2003). The first three levels are as follows (Gutiérrez, Jaime & Fortuny, 1991):

1. At *Level 1 (Recognition)*, students recognise and name solids, and distinguish solids from each other on a visual basis.

2. At *Level 2 (Analysis)*, students identify the components of solids and can discover properties of the solids by experimentation.

3. At *Level 3 (Informal Deduction)*, students logically order properties of solids, and understand definitions (necessary and sufficient properties) and class inclusions.

Characteristics of the Levels

Van Hiele identified five characteristics of the levels (van Hiele, 1959/1984):

1. *Sequential.* The levels are sequential, implying that students require adequate and effective learning experiences at lower levels in order to learn how to think and reason at higher levels.

2. *Intrinsic and extrinsic.* Geometric concepts that are implicitly understood at one level become explicitly understood at the next level.

3. *Linguistics.* Each level has its own language, set of symbols and network of relations.

4. *Mismatch.* If students are at one level and the teacher, instructional materials, content, and vocabulary are at a higher level than the students, then students may not learn and progress as much as we would like, because they will not be able to understand the thought processes being used.

5. *Advancement.* According to van Hiele (1986: 50), "the transition from one level to the following is not a natural process; it takes place under the influence of a teaching-learning program."

Phases of Learning

To help students progress from one level to the next, the van Hieles propose a sequence of five phases of learning, or "phase-based instruction" (Hoffer & Hoffer, 1992; van Hiele, 1959/1984, 1986; van Hiele-Geldof, 1959/1984):

Phase 1: Information. The teacher engages the students in conversation about the topic of study, evaluates their responses, learns how they interpret the words used and gives them some awareness of why they are studying the topic, so as to set the stage for further study.

Phase 2: Guided orientation. Next, students actively explore the topic of study by doing short (often one-step) tasks designed to elicit specific responses. These steps help students acquaint themselves with the objects from which geometric ideas are abstracted.

Phase 3: Explicitation. In this phase, students learn to express their opinions about the structures observed during class discussions. The teacher leads students' discussion of the objects of study in their own words, so that students become explicitly aware of the objects of study. Then, the teacher introduces the relevant vocabulary.

Phase 4: Free orientation. Next, the teacher challenges students with more complex tasks that can be completed in different ways. The teacher encourages students to solve and elaborate on these problems and their solution strategies.

Phase 5: Integration. In this final phase, students summarise what they have learned about the objects of study with the goal of creating an overview of the topic. The teacher guides students through this process using standard vocabulary, but does not present any new ideas. At the completion of this phase, the students should have attained a new level of thinking about the topic of study.

METHODOLOGY

I used a case study research design and purposeful sampling to select the case study participants. The case selection criteria were: (a) One class of Form One students studying in a public academic secondary school with a well-equipped computer laboratory; (b) The class was of mixed mathematical and English language ability (according to achievement in the 2006 school mid-year examination); (c) The students were of mixed gender and race; and (d) They had not learned Chapter 12 (Solid Geometry) in school. The criteria for selecting the students from this class to participate in the study were: (a) Two students from each mathematical achievement level (that is two low-ability, two average-ability and two high-ability students) who had different English language achievement levels in the 2006 school mid-year examination; and (b) They volunteered to serve as case study participants (with their parents' consent). Table 1 shows the participants' achievement levels in Mathematics and English language in the 2006 school mid-year examination.

Table 1. Participants' Mathematics and English language achievement levels

Participant	2006 school mid-year examination	
	Mathematics achievement	English achievement
Jeff	Low	Low
Niki	Low	Average
Farah	Average	Low
Sharmini	Average	Average
Enn	High	Average
Yee	High	High

Procedure

The study comprised three sessions (see Figure 2). During Session 1, I did a pre-interview with each individual participant prior to phase-based instruction with GSP, in order to determine their initial van Hiele levels of geometric thinking about cubes and cuboids. All the interviews were videotaped. During Session 2, we taught the class about properties of cubes and cuboids through the van Hieles' phase-based instruction with GSP. The session comprised seven 40-minute lessons and 14 phase-based GSP instructional activities. During Session 3, I did a post-interview with each individual participant in order to determine their van Hiele level after phase-based instruction with GSP. All the interviews were also videotaped.

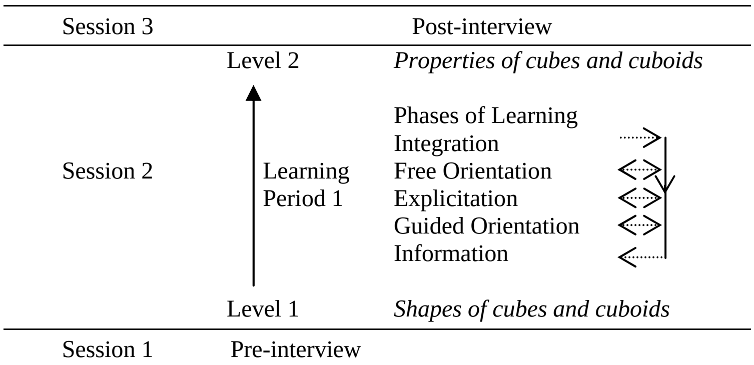

Figure 2. Three sessions of the study

Data Analysis

Videotapes of pre- and post-interviews were transcribed verbatim and each participant's verbal transcript was analysed and scored according to Mayberry's scoring criteria (1981) and the classification of the degrees of acquisition of van Hiele levels by Gutierrez et al. (1991). Next, I compared and contrasted the verbal transcripts of each participant's pre- and post-interviews in order to determine the changes in their van Hiele levels. To determine inter-coder reliability, along with a local public university mathematics teacher, we coded the data and computed Cohen's Kappa, using SPSS version 13.0 for Windows. Table 2 shows the Kappa values for cubes and cuboids in the pre- and post-interviews. The high Kappa values indicated that the reliability of the assignment of the van Hiele levels about cubes and cuboids was adequate (Krippendorf, 1980 and Gottschalk & Bechtel, 1993, as cited in Bernard, 2000).

Table 2. Summary of Kappa values

Inter-coder reliability	Pre-interview		Post-interview	
	Cubes	Cuboids	Cubes	Cuboids
Kappa	.92	.91	.87	.93

FINDINGS AND DISCUSSION

Initial van Hiele Levels

Cubes

The participants' initial van Hiele levels of geometric thinking about cubes fell into three categories (see Table 3): 'Level 0 thinker,' 'Level 1 thinker' and 'Level 2 thinker.' Niki, the 'Level 0 thinker' had an ordered triple of (0, 0, 0) which implied that he was at Level 0 with no acquisition of Levels 1, 2 and 3. He could not recognise and name a cube and could not discriminate cubes from cuboids.

The 'Level 1 thinker' had two sub-categories: 'Low Level 1 thinker' and 'High Level 1 thinker.' Jeff, the 'Low Level 1 thinker', had an ordered triple of (1_L, 0, 0). He was at Level 1, with low acquisition of Level 1 and no acquisition of Levels 2 and 3. Jeff could recognise and name a cube, but could not discriminate cubes from cuboids. Sharmini, the 'High Level 1 thinker', had an ordered triple of (1_H, 0, 0). She was at Level 1, with high acquisition of Level 1 and no acquisition of Levels 2 and 3. Sharmini could recognise and name a cube as well as discriminate the cubes from the cuboids.

There were three sub-categories of ‘Level 2 thinker’: ‘Low Level 2 thinker,’ ‘High Level 2 thinker’ and ‘High Level 2[a] thinker.’ Farah, the ‘Low Level 2 thinker’, had an ordered triple of $(1_H, 1_L, 0)$. She was at Level 2, with high acquisition of Level 1, low acquisition of Level 2 and no acquisition of Level 3. Farah could recognise and name a cube, discriminate cubes from cuboids, and identify the property of vertices and one property of faces. But she could not identify any properties of the edges of a cube. Enn, the ‘High Level 2 thinker’, had an ordered triple of $(1_H, 1_H, 0)$. She was at Level 2 with high acquisition of Levels 1 and 2 and no acquisition of Level 3. Enn could recognise and name a cube, discriminate cubes from cuboids, and identify the property of vertices and one property of the edges and faces of a cube. Yee, the ‘High Level 2[a] thinker’, had an ordered triple of $(1_L, 1_H, 0)$[a]. She was at Level 2, with low acquisition of Level 1 but high acquisition of Level 2 and no acquisition of Level 3. Yee could recognise and name a cube but could not discriminate cubes from cuboids. Nevertheless, she could identify the property of vertices and one property of the edges and faces of a cube. Even though her degrees of acquisition of the first two levels did not follow a decreasing order (i.e., the higher the level, the lower the degrees of acquisition), her ordered triple fit the hierarchical structure of the van Hiele levels: that is, she attained Levels 1 and 2 in order to be assigned Level 2 (Gutiérrez et al., 1991).

Table 3. Participants’ initial Van Hiele levels of geometric thinking about cubes

Participant	Mathematics Achievement	English Achievement	Category	Sub-category	Ordered Triple	Level	Degrees of Acquisition
Niki	Low	Average	Level 0 thinker	Level 0 thinker	$(0, 0, 0)$	Level 0	No Level 1, No Level 2, No Level 3
Jeff	Low	Low		Low Level 1 thinker	$(1_L, 0, 0)$	Level 1	Low Level 1, No Level 2, No Level 3
Sharmini	Average	Average	Level 1 thinker	High Level 1 thinker	$(1_H, 0, 0)$	Level 1	High Level 1, No Level 2, No Level 3
Farah	Average	Low		Low Level 2 thinker	$(1_H, 1_L, 0)$	Level 2	High Level 1, Low Level 2, No Level 3
Enn	High	Average	Level 2 thinker	High Level 2 thinker	$(1_L, 1_H, 0)$[a]	Level 2[a]	Low Level 1, High Level 2, No Level 3
Yee	High	High		High Level 2[a] thinker	$(1_L, 1_H, 0)$[a]	Level 2[a]	Low Level 1, High Level 2, No Level 3

[a]The ordered triple fitted the hierarchical structure of the van Hiele levels, but the degrees of acquisition of the levels did not follow a decreasing order.

Cuboids

Similarly, the participants' initial van Hiele levels of geometric thinking about cuboids fell into three categories (see Table 4): 'Level 0 thinker,' 'Level 1 thinkers' and 'Level 2 thinker.' Niki and Jeff, the 'Level 0 thinker', had an ordered triple of (0, 0, 0). They were at Level 0, with no acquisition of Levels 1, 2 and 3. Niki and Jeff could not recognise or name a cuboid, and could not discriminate the cuboids from the rhomboid and parallelepipeds.

Farah and Sharmini, the 'Level 1 thinker' had an ordered triple of (1_L, 0, 0). They were at Level 1, with low acquisition of Level 1 and no acquisition of Levels 2 and 3. Farah and Sharmini could recognise and name a cuboid, but could not discriminate the cuboids from the rhomboid and parallelepipeds (Low Level 1 thinkers).

The 'Level 2 thinker' had two sub-categories: 'Low Level 2 thinker' and 'High Level 2 thinker.' Yee, the 'Low Level 2 thinker', had an ordered triple of (1_L, 1_L, 0). She was at Level 2, with low acquisition of Levels 1 and 2 and no acquisition of Level 3. Yee could recognise and name a cuboid, but could not discriminate the cuboids from the rhomboid and parallelepipeds. Further, she could identify two properties of edges and faces, but could not identify the property of the vertices of a cuboid. Enn, the 'High Level 2 thinker', had an ordered triple of (1_H, 1_H, 0). She was at Level 2, with high acquisition of Levels 1 and 2 and no acquisition of Level 3. Enn could recognise and name a cuboid, discriminate the cuboids from the rhomboid and parallelepipeds, and identify the property of vertices and one property of the edges and faces of a cuboid.

Table 4. Participants' initial Van Hiele levels of geometric thinking about cuboids

Participant	Mathematics Achievement	English Achievement	Category	Sub-category	Ordered Triple	Level	Degrees of Acquisition
Niki	Low	Average	Level 0 thinkers	Level 0 thinkers	(0, 0, 0)	Level 0	No Level 1, No Level 2, No Level 3
Jeff	Low	Low			(0, 0, 0)	Level 0	No Level 1, No Level 2, No Level 3
Sharmini	Average	Average	Level 1 thinkers	Low Level 1 thinkers	(1_L, 0, 0)	Level 1	Low Level 1, No Level 2, No Level 3
Farah	Average	Low			(1_L, 0, 0)	Level 1	Low Level 1, No Level 2, No Level 3
Yee	High	High	Level 2 thinkers	Low Level 2 thinker	(1_L, 1_L, 0)	Level 2	Low Level 1, Low Level 2, No Level 3
Enn	High	Average		High Level 2 thinker	(1_H, 1_H, 0)	Level 2	High Level 1, High Level 2, No Level 3

Table 5. Changes in the participants' Van Hiele levels of geometric thinking about cubes after phase-based instruction with GSP

Participant (Maths & Eng. Achievement; Initial Levels)	Category	Sub-category	Before Phase-Based Instruction			After Phase-Based Instruction			Changes		
			Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition
Niki (Low, Average; Level 0)	Progressed from L0 to L2	Progressed from L0 to high L2	(0, 0, 0)	Level 0	No Level 1, No Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(+1_{+2}, +1_{+2}, 0_0)$	+2	+2 +20
Jeff (Low, Low; Level 1)	Progressed from L1 to L2	Progressed from low L1 to high L2	$(1_L, 0, 0)$	Level 1	Low Level 1, No Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_{+1}, +1_{+2}, 0_0)$	+1	+1 +20
Sharmini (Average, Average; Level 1)		Progressed from high L1 to high L2[a]	$(1_H, 0, 0)$	Level 1	High Level 1, No Level 2, No Level 3	$(1_L, 1_H, 0)$[a]	Level 2[a]	Low Level 1, High Level 2, No Level 3[a]	$(0_{-1}, +1_{+2}, 0_0)$	+1	–1 +20

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Table 5. *(continued)*

Participant (Maths & Eng. Achievement; Initial Levels)	Category	Sub-category	Before Phase-Based Instruction			After Phase-Based Instruction			Changes		
			Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition
Farah (Average, Low; Level 2)		Progressed from low L2 to high L2	$(1_H, 1_L, 0)$	Level 2	High Level 1, Low Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_0, 0_{+1}, 0_0)$	0	0 +10
Yee (High, High; Level 2)	Remained at L2	Progressed from high L2[a] to high L2	$(1_L, 1_H, 0)$[a]	Level 2[a]	Low Level 1, High Level 2, No Level 3[a]	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_{+1}, 0_0, 0_0)$	0	+100
Enn (High, Average; Level 2)		Remained at high L2	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_0, 0_0, 0_0)$	0	000

[a]The ordered triple fit the hierarchical structure of the van Hiele levels but the degrees of acquisition of the levels did not follow a decreasing order.

Changes in van Hiele Levels

Cubes

Changes in the participants' van Hiele levels of geometric thinking about cubes after phase-based instruction with GSP fell into three categories (see Table 5). The categories were 'Progressed from L0 to L2,' 'Progressed from L1 to L2' and 'Remained at L2.' Niki, who belonged to the first category, progressed from Level 0 (with no acquisition of Levels 1, 2 and 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Thus, there was a positive change in Levels 1 and 2 and a positive change of two degrees of acquisition of Levels 1 and 2.

The second category had two sub-categories: 'Progressed from low L1 to high L2' and 'Progressed from high L1 to high L2[a].' Jeff (the first sub-category) progressed from Level 1 (with low acquisition of Level 1 and no acquisition of Levels 2 and 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Thus, there was a positive change of Level 2 and a positive change of one and two degrees of acquisition of Levels 1 and 2 respectively. Sharmini (the second sub-category) progressed from Level 1 (with high acquisition of Level 1 and no acquisition of Levels 2 and 3) to Level 2 (with low acquisition of Level 1, high acquisition of Level 2 and no acquisition of Level 3). There was a positive change of Level 2 as well as a negative change of one degree of acquisition of Level 1 and a positive change of two degrees of acquisition of Level 2. Further, her ordered triple for cubes after phase-based instruction with GSP fit the hierarchical structure of the van Hiele levels, but her degrees of acquisition of the levels did not follow a decreasing order.

The third category had three sub-categories: 'Progressed from low L2 to high L2,' 'Progressed from high L2[a] to high L2' and 'Remained at high L2.' Farah (the first sub-category) progressed from Level 2 (with high acquisition of Level 1, low acquisition of Level 2 and no acquisition of Level 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Hence, there was no change in level but a positive change of one degree of acquisition of Level 2. Yee (the second sub-category) progressed from Level 2 (with low acquisition of Level 1, high acquisition of Level 2 and no acquisition of Level 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Therefore, there was no change in level but a positive change of one degree of acquisition of Level 1. Enn (the third sub-category) remained at Level 2 with high acquisition of Levels 1 and 2 and no acquisition of Level 3 before and after Learning Period 1. Thus, there were no changes in level or degrees of acquisition of the levels.

Cuboids

Likewise, changes in the participants' van Hiele levels of geometric thinking about cuboids after phase-based instruction with GSP fell into three categories (see Table 6). They were 'Progressed from L0 to L2,' 'Progressed from L1 to L2' and 'Remained at L2.' The first category had two sub-categories: 'Progressed from L0 to low L2' and 'Progressed from L0 to high L2.' Jeff (the first sub-category) progressed from Level 0 (with no acquisition of Levels 1, 2 and 3) to Level 2 (with high acquisition of Level 1, low acquisition of Level 2 and no acquisition of Level 3). Hence, there was a positive change of Levels 1 and 2 and a positive change of two and one degrees of acquisition of Levels 1 and 2, respectively. Niki (the second sub-category) progressed from Level 0 (with no acquisition of Levels 1, 2 and 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Therefore, there was a positive change in Levels 1 and 2 and a positive change of two degrees of acquisition of Levels 1 and 2, respectively.

The second category also had two sub-categories: 'Progressed from low L1 to high L2[a]' and 'Progressed from low L1 to high L2.' Sharmini (the first sub-category) progressed from Level 1 (with low acquisition of Level 1 and no acquisition of Levels 2 and 3) to Level 2 (with low acquisition of Level 1, high acquisition of Level 2 and no acquisition of Level 3). Thus, there was a positive change in Level 2 and a positive change of two degrees of acquisition of Level 2. Her ordered triple for cuboids fit the hierarchical structure of the van Hiele levels, but her degrees of acquisition of the levels did not follow a decreasing order. Farah (the second sub-category) progressed from Level 1 (with low acquisition of Level 1 and no acquisition of Levels 2 and 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Hence, there was a positive change of Level 2 and a positive change of one and two degrees of acquisition of Levels 1 and 2.

The third category also had two sub-categories: 'Progressed from low L2 to high L2' and 'Remained at high Level 2.' Yee (the first sub-category) progressed from Level 2 (with low acquisition of Levels 1 and 2 and no acquisition of Level 3) to Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3). Thus, there was no change in Level 2 but a positive change of one degree of acquisition of Levels 1 and 2. Enn (the second sub-category) remained at Level 2 (with high acquisition of Levels 1 and 2 and no acquisition of Level 3) before and after Learning Period 1 for cuboids. Hence, there was no change in level and no change in degree of acquisition of the levels.

Table 6. Changes in the participants' van Hiele levels of geometric thinking about cuboids after phase-based instruction with GSP

Participant (Maths & Eng. Achievement; Initial Levels)	Category	Sub-category	Before Phase-Based Instruction			After Phase-Based Instruction			Changes		
			Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition
Jeff (Low, Low; Level 0)	Progressed from L0 to L2	Progressed from L0 to low L2	(0, 0, 0)	Level 0	No Level 1, No Level 2, No Level 3	$(1_H, 1_L, 0)$	Level 2	High Level 1, Low Level 2, No Level 3	$(+1_{+2}, +1_{+1}, 0_0)$	+2	+2 +1 0
Niki (Low, Average; Level 0)		Progressed from L0 to high L2	(0, 0, 0)	Level 0	No Level 1, No Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(+1_{+2}, +1_{+2}, 0_0)$	+2	+2 +2 0

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Table 6. *(continued)*

Participant (Maths & Eng. Achievement; Initial Levels)	Category	Sub-category	Before Phase-Based Instruction			After Phase-Based Instruction			Changes		
			Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition
Sharmini (Average, Average; Level 1)	Progressed from L1 to L2	Progressed from low L1 to high L2[a]	$(1_L, 0, 0)$	Level 1	Low Level 1, No Level 2, No Level 3	$(1_L, 1_H, 0)$[a]	Level 2[a]	Low Level 1, High Level 2, No Level 3[a]	$(0_0, +1_{+2}, 0_0)$	+1	0 +20
Farah (Average, Low; Level 1)		Progressed from low L1 to high L2	$(1_L, 0, 0)$	Level 1	Low Level 1, No Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_{+1}, +1_{+2}, 0_0)$	+1	+1 +20

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Table 6. *(continued)*

Participant (Maths & Eng. Achievement; Initial Levels)	Category	Sub-category	Before Phase-Based Instruction			After Phase-Based Instruction			Changes		
			Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition	Ordered Triple	Level	Degrees of Acquisition
Yee (High, High; Level 2)	Remained at L2	Progressed from low L2 to high L2	$(1_L, 1_L, 0)$	Level 2	Low Level 1, Low Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_{+1}, 0_{+1}, 0_0)$	0	+1 +10
Enn (High, Average; Level 2)		Remained at high L2	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(1_H, 1_H, 0)$	Level 2	High Level 1, High Level 2, No Level 3	$(0_0, 0_0, 0_0)$	0	000

[a]The ordered triple fit the hierarchical structure of the van Hiele levels but the degrees of acquisition of the levels did not follow a decreasing order.

CONCLUSION

These findings suggest several ways for Form One mathematics teachers to improve students' van Hiele levels of geometric thinking about cubes and cuboids. First, teachers need to organise sequences of lessons comprising well-designed instructional activities that move very deeply through the levels of geometric thinking and the five phases of learning, not only to enrich students' thinking at the current level but also to move them toward the next level in order to develop a deeper understanding of the concepts.

Second, teachers need to use GSP appropriately based on students' van Hiele levels to avoid mismatches between levels: that is, students at van Hiele Level 1 have difficulty constructing models of cubes and cuboids in GSP because they do not yet know their properties (that is, Level 2) (de Villiers, 1999). Further, the pre-constructed GSP models prevented the Level 1 thinkers from getting bogged down in constructing the models themselves (which is inappropriate for their level), letting them focus on how to analyse the models' properties instead. For example, by directly manipulating the GSP model of a cuboid to generate many examples of cuboids, the students were able to recognise its shape and understand that cuboids always have equal opposite edges by analysing the measurement of its edges. Through their actions (dynamic manipulation) and reflecting on those actions, students were able to understand properties of cubes and cuboids.

Third, teachers need to know their students' levels of geometric thinking and the content areas they are teaching, and also have adequate resources to support their work so that they can serve in the various roles competently throughout all the five phases of learning.

All in all, these essential components of the phase-based instructional environment using GSP helped improve students' van Hiele levels of geometric thinking about cubes and cuboids. This suggests for this sample that with well-designed instructional activities, appropriate tools, and teacher guidance, students can learn important solid geometric concepts with increasing understanding.

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