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A STUDY OF MALAYSIAN YEAR 5 PUPILS' PRE-ALGEBRAIC THINKING

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Abstract. This paper reports part of the findings of a study undertaken to investigate Malaysian Year 5 pupils' pre-algebraic thinking while they were solving pre-algebraic problems. It focuses on the inference of pre-algebraic thinking among five 11-year-old pupils while solving three pre-algebraic problems involving geometric patterns. Data for this study was collected on a one-by-one basis, during which each participant was observed while solving the problems. While they were solving the problems, the participants were required to think aloud by verbalising whatever came to mind during the problem-solving process. After solving each problem, retrospective interviews were conducted if there was reason to think that additional information could be elicited from the participant. The think-aloud and interview sessions were audio-taped and transcribed. The verbatim transcripts were given to the participants to achieve a negotiated interpretation of the data. The verbatim transcripts were then segmented to enable inferences to be made about the participants' pre-algebraic thinking. Analysis of the data suggested that 'look for pattern', 'recognise pattern' and 'extend pattern' were among the types of common pre-algebraic thinking inferred from the participants' verbalisations while solving problems involving geometric patterns.

Keywords: pre-algebraic thinking, geometric patterns

INTRODUCTION

The inability of many Malaysian secondary school students to master algebraic concepts and skills has been documented by the Malaysian public examination performance reports for the subject of mathematics at both the lower and upper secondary levels (Lembaga Peperiksaan, 1995, 1996a, 1996b, 1997a, 1997b, 2002). This seems to suggest an urgent need to consider the preparation for pre-algebra in Malaysian primary schools. However, little effort has been made to look into or evaluate Malaysian primary school pupils' emerging abilities in algebra. With the recent awareness of the need to develop algebraic thinking in the early grades, there is a need to explore and understand pre-algebraic thinking among Malaysian primary school pupils who are not exposed to algebra formally and directly in their classroom.

This paper reports part of the findings of a study undertaken to investigate the pre-algebraic thinking underlying the solving of pre-algebraic problems among 11-year-old Year 5 pupils in some suburban primary schools in the Division of Kota Samarahan, Sarawak. The study focused on pre-algebraic thinking inferred from five participants while they were solving pre-algebraic problems concerning geometric patterns. Therefore, the following research question is addressed in this paper:

> What pre-algebraic thinking is inferred from five primary school Year 5 pupils in solving pre-algebraic problems concerning geometrical patterns?

REVIEW OF RELATED LITERATURE

Emerging Algebraic Thinking

There are various views of how algebraic thinking emerges. Mason (1996) pointed out that the roots of algebraic thinking may be traced to the processes of (a) detecting sameness and differences, (b) making distinctions, (c) classifying and labeling, and (d) algorithm seeking. On the other hand, Slavit (1999) claimed that algebraic thinking develops with numeric and arithmetic understanding, while Dr. Bill Jacobs (in California State Board of Education [CSBE], 2000) explained that algebraic ideas emerge as students create, discuss, recognise, describe, represent and extend patterns. Blanton and Kaput (2003) also related algebraic thinking to activities such as looking for patterns.

Patterns and Algebraic Thinking

Patterns are a "regular occurrence in mathematics" (Van de Walle, 2001, p. 384). Patterns can be recognised, extended and generalised, which is an important process in algebraic thinking. In mathematics, patterns can be found in numbers as well as in geometric situations.

Elementary school children have tremendous intellectual curiosity regarding patterns (Plager, Klinger, & Rooney, 1997). As such, children should be encouraged to engage in algebraic activities such as recognising, describing, extending and creating a wide variety of patterns. Burns (2002) also stressed patterns as an algebraic concept that can be developed early. She proposed that children could learn to recognise, extend, create and generalise growth patterns. Ideas proposed by Burns seem to reinforce the ideas of Urquhart (2000), who suggested that recognising patterns was an algebraic skill that can be developed early.

Patterning requires students not only to identify, analyse, describe, create and extend patterns, but also to draw appropriate generalisations (Warren, 2000b). This follows from the fact that the ability to reason visually is significantly correlated with most early algebraic experiences, especially when generalising from visual patterns. The process of generalising and justifying patterns at the level of early algebra requires students to (a) produce additional examples of the same kind, (b) use an evolving pattern in a given situation, (c) generalise an observed pattern, and (d) justify conclusions (Friedlander & Hershkowitz, 1997, p. 443).

Pre-algebraic Thinking

Early algebra is not about introducing traditional, formal algebra into primary schools, but instead is about developing arithmetical reasoning in conjunction with algebraic reasoning (Warren, 2002). Boero (2001) called this 'pre-algebra'. Some of the core ideas of pre-algebra can be introduced to primary school pupils, for example, recognising patterns (Kutz, 1991) and generalising and justifying patterns (Friedlander & Hershkowitz, 1997).

Algebraic experience in elementary schools is essential to building the thinking that is "an important precursor to the more formalised study of algebra in the middle and secondary schools" (National Council of Teachers of Mathematics [NCTM], 2000, p. 159). This reasoning suggests that the basics of algebraic thinking may develop from arithmetic thinking and then be transformed into algebraic thinking. Warren (2000a) called this transition of thinking from arithmetic to algebraic thinking pre-algebraic thinking. 'Pre-algebraic' refers to transformations that happen without or before algebraic formalisation and further implies that pre-algebra only involves transformation through arithmetic, geometric or physical manipulation of variables such as adding, subtracting, translating and equilibrating (Boero, 2001).

To develop their pre-algebraic thinking, young children must be engaged in generalisation and algebraic activities (Lee, 2001) such as making general statements about shapes or geometric patterns. Thus, the study of patterns may be a productive way of developing algebraic reasoning (Femini-Mundy, Lappan, & Phillips, 1997) and the ability to think algebraically (Kenney & Silver, 1997) in the elementary grades.

Developing an awareness of generality and applying it in the mathematical domain is itself an indicator of algebraic thinking (Irwin & Britt, 2005). Driscoll and Moyer (2001) constructed a guideline with five indicators of algebraic thinking, namely (a) systematically searching for a rule, (b) forming a generalized rule, (c) conjecturing a generalised rule, (d) representing this rule in other forms, and (e) connecting the different representations. Thus, pre-algebraic thinking is a process or an action (NCTM, 2004) that may be detected by some indicators.

Conceptual Framework of Pre-algebraic Thinking

Based on a review of the literature on how algebraic thinking emerges (e.g., Mason, 1996; Slavit, 1999; CSBE, 2000), patterning processes (e.g., Friedlander & Hershkowitz, 1997; Warren, 2000a; Burns, 2002), the nature of pre-algebraic thinking (e.g., Warren, 2000a; 2002; Boero, 2001) and indicators of algebraic thinking (e.g., Driscoll & Moyer, 2001; Irwin & Britt, 2005), a conceptual framework for pre-algebraic thinking was constructed (Table 1). This framework enabled the researchers to infer the pupils' pre-algebraic thinking based on some indicators related to what they said and did while solving the problems.

Table 1. Pre-algebraic thinking framework

Pre-algebraic thinking	Indicator
Look for pattern	Find difference between consecutive terms
Recognise pattern	State difference between consecutive terms
Describe pattern	Explain how a pattern grows
Generalise pattern	Make general statement about the pattern
Extend pattern	Produce additional term
Algorithm seeking	State operation to be used
Justify generalisation /conclusion	Explain the generalisation or conclusion made
Test conjecture	Generate and evaluate conjecture
Use multiple representations	Involve more than one single mode of representation in solution process

METHODOLOGY

Research Design

The research design adopted was based on cognitive task analysis (CTA). According to Chipman, Schraagen and Shalin (2000, p. 3), "CTA is the extension of traditional task analysis techniques to yield information about the knowledge, thought processes, and goal structures that underlie observable task performance". From this perspective, CTA is very much related to knowledge elicitation. Cooke (1994) proposed 'process tracing' as one of several knowledge elicitation techniques. Process tracing uses verbal reports to make inferences about the cognitive processes underlying task performance, which means that the researchers must interpret the participants' thinking. A common method for interpreting human thinking is verbal protocol analysis (VPA), described by

Crutcher (1994) as a core method for analysing thinking. Mulhern (1989) also highlighted the success of verbal protocol analysis to elicit appropriate forms of intellectual activities through problem solving, as it required taking as complete a record as possible of what an individual did to solve a problem.

Participants

Primary school Year 5 pupils were chosen as the study participants for several reasons. First, Year 5 students can be expected to develop basic algebraic thinking (Cai, 2001). Moreover, Year 5 pupils generally possess a greater knowledge base (Siegler, 1979) from which new knowledge can be derived, especially if they must use their prior experience and knowledge of mathematics to solve pre-algebraic problems to which they have not been formally and directly exposed in their Mathematics lessons in Malaysia. Year 5 pupils also may be assumed to have the necessary language and communication skills to be able to think aloud and explain their solution processes verbally during interviews, as verbal protocols are most readily obtained from those who are fluent and confident verbalisers (Bryne, 1983).

Instrumentation

Several factors were considered in the choice of pre-algebraic problems used in this study. First, the problems had to involve participants in looking for, recognising and extending patterns, which can be related to pre-algebraic thinking (Table 1). Second, the problems had to enable the participants to easily think aloud. This meant that the problems had to be well-specified and not demand very abstract reasoning or long chains of deduction. Third, the problems had to be within the reach of children in the concrete-operations stage (Siegler, 1979) as primary school pupils were involved in the study. The three pre-algebraic problems are presented together with the analysis of data.

Methods of Data Collection

This paper focuses on verbal data collected via think aloud verbalisations and interviews.

Think aloud verbalisation

This method of data collection was used to determine the participants' pre-algebraic thinking (Redding, 1995; Pugalee, 2004) via process tracing (Cooke, 1994) and verbal protocol analysis (Bryne, 1983; Crutcher, 1994). The pre-algebraic problems were presented to the participants along with verbalisation instructions and audio recording.

To ensure a continuous verbal flow of the participants' thinking, participants were prompted to say as much as possible by responding to the question: "What are you thinking right now?" When the participant fell into silence, he or she was prompted to keep talking by saying, "Please think aloud". To minimise interruptions, participants were prompted only if they remained silent for more than five seconds. If a participant gave an incorrect solution, no feedback was given.

Interview

Interviews were used to collect supplementary verbal data. These interviews involved retrospective questioning to supplement concurrent think-aloud verbalisations if there was reason to think that additional information could be elicited from the participant, especially when the concurrent verbalisations were deemed incomplete. During the interview, the participants' thinking was 'pressed' through the question: "How did you arrive at that answer?". The responses provided greater insights into how the participants conceptualised the mathematics and clarified their thinking processes.

Credibility of the Data

Credibility refers to the internal validity of the data (Mertems, 1998; Maxwell, 2005). Member checks were used to achieve the credibility of the data. A member check involved each participant reviewing his or her transcript and confirming the accuracy or corrected misinterpretations that had been included in the transcriptions.

STEPS IN DATA ANALYSIS

We went through the following steps in analysing the verbal data:

Step 1 – Organise and Prepare the Data for Analysis

All of the verbal data from the participants' think-aloud and interview protocols were transcribed verbatim. In the transcript, participants' think-aloud statements were written in the form of a text, and the interview protocols were presented in the form of conversation between the participant and the researcher. Syntactic breaks in the participants' think-aloud and conversations were indicated by commas, periods or semicolons, as appropriate. Pauses of short duration of less than five seconds were indicated by ellipses (…), whereas pauses of more than five seconds were indicated by the word "pause" within parentheses.

Step 2 – Read Through All the Transcripts

We read through all the transcripts to identify segments or phrases that were related to or reflected the pre-algebraic thinking indicators, as shown in Table 1. All these segments or phrases were underlined.

Step 3 – Making Inference

All the verbatim transcripts were broken down into short segments or phrases to be inferred. A sentence formed the basic unit for a segment unless it contained more than one idea, in which case it was further segmented. The process of making inference illustrated the existence of pre-algebraic thinking based on the conceptual framework, as shown in Table 1.

Step 4 – Confirmability of Data Analysis

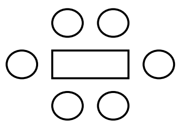
Two experienced researchers in the field of mathematics education evaluated at least three verbal transcripts for each problem to verify that the inferred pre-algebraic thinking 'fit' the data and that particular data had been properly 'fitted' into particular pre-algebraic thinking. This served to reduce researcher-bias, thus contributing towards objectivity in data analysis.

ANALYSIS OF DATA

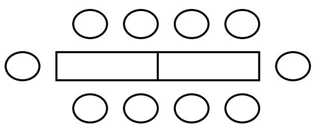
Results of analysis are presented based on problems. The problem is presented followed by analysis of verbal protocols collected via think-aloud verbalisations and/or interviews.

The 'Table' Problem (Adapted from Kaput & Blanton, 2001)

Andrew is setting up tables for a birthday party. He knows that six people can sit about this table.



When he puts two of these tables together end to end, he can seat ten people.



How many people can Andrew seat if he puts three tables together end to end? Show and explain how you found your answer.

This problem involves a regular geometric pattern in which the pattern grows in a 'fixed' way. All five participants solved this problem successfully (Table 2).

Table 2. Verbal protocol analysis for the 'Table' problem

Participant	Pre-algebraic thinking inferred	Verbal protocols	Source of verbal protocols
Ali	Look for pattern	"So one table has 6 people to sit around. Two tables have 10 people."	Think-aloud
	Extend pattern	"So 3 tables…1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14."	Think-aloud
Zita	Extend pattern	"I will join 3 tables end to end and I will place the chairs…1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14."	Think-aloud
	Recognise pattern	"Because the other diagrams have 2 (persons sitting on the side)."	Interview
Rina	Extend pattern	"To start off, I draw…3 tables…1, 2, 3. (Pause) Then I will think how many people can be seated. One at this end, 1 at the other end. 2 here, 2 here, 2 here, 2 here, 2 here, 2 here and have…1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 people can sit around 3 tables joined end to end."	Think-aloud
	Recognise pattern	"Because as in given diagrams, only 1 person sits at the end. So I draw 1 person sitting at the end. The same goes to 2 persons sitting at the sides."	Interview
Firul	Extend pattern	"10…14 persons."	Think-aloud
	Look for pattern	"Because at start there are 6 persons. When 2 tables are joined end to end, 10 persons can be seated. So I count through 6 added to what gives 10"	Interview
	Algorithm seeking	"…6, 7, 8, 9, 10…got 4. 10 I added to 4 gives the answer 14 persons."	Interview
Diana	Look for pattern	"The first table has 6 chairs. The second diagram has 10 chairs."	Think-aloud
	Test conjecture	"In the first diagram got 6 chairs and 6 is in the times-2-table. 2 times 3 is 6. The second diagram is also related to Times-2-table. 2 times 5 is 10."	Think-aloud
	Extend pattern	"1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14."	Think-aloud
	Justify generalisation	"So we try to look into times-2-table. Is there an answer 14? Times-2-table has the answer 14. 14 divided by 2 is 7, so 7 times 2 is 14. So 3 tables have 14 chairs."	Think-aloud
	Recognise pattern	"I follow the second diagram."	Interview

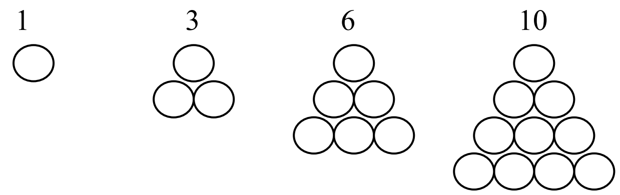
Ali, Firul and Diana tried to look for a pattern by using the two diagrams given (Table 2). Ali could extend the pattern immediately by drawing, whereas Firul used some mental arithmetic to help him arrive at the answer without drawing a diagram. His use of mental computation led to the inference of 'algorithm seeking'.

Diana related the number of chairs to multiples of 2, which later led her to justify her generalization by verifying whether 14 is a multiple of 2 before concluding her answer. Her strategy of relating the number of chairs to multiples of 2 and verification of her answer against the 'Times-2-table' led to the inference of 'test conjecture' and 'justify generalization' from her verbal protocols. Diana then extended the pattern via drawing. During the retrospective interview, she seemed to describe the pattern based on the second diagram only.

Rina and Zita also extended the pattern via drawing. During the interview, Rina and Zita seemed to extend the pattern based on their ability to recognise the pattern as verbalised.

The "Triangular Number' Problem (Adapted from Pugalee, 2004)

The diagram below shows the first four triangular numbers.



What is the fifth triangular number? Explain how you find the answer.
What is the seventh triangular number? Explain how you find the answer.

This problem involves an irregular geometric pattern where the pattern 'grows' in an increasing but generalisable way. For this problem, only Ali, Zita, Rina and Firul's verbal protocols were analysed, as Diana could not solve the problem successfully (Table 3).

Table 3. Verbal protocol analysis for the 'Triangular Number' problem

Participant	Pre algebraic thinking inferred	Verbal protocols (translated from the Malay language)	Source of verbal protocols
Ali	Look for pattern	"I try to find the distance between the numbers. Distance between 1 and 3 is 2. Distance between 6 and 10 is 4. So the distances are not the same."	Think-aloud
	Describe pattern	"So if the distance between (1 and 3) is 2, distance between (3 and 6) is 3, distance between (6 and 10) is 4, then the distance to the 5th number is 5."	Think-aloud
	Extend pattern Algorithm seeking	"So 10 plus 5, 15. So 15 is the (5th triangular) number."	Think-aloud
	Extend pattern Algorithm seeking	"15 plus 6, 21. 21 plus 7, 28. So the answer is 28."	Think-aloud
Zita	Look for pattern	"I will try 10 minus 6. The answer is 4."	Think-aloud
	Recognise pattern	"6 minus 3, the answer is 3. Three minus 1, the answer is 2. So I get the answer."	Think-aloud
	Extend pattern Algorithm seeking	"10 plus 5 equals 15. So the 5th triangular number has 15 circles."	Think-aloud
	Extend pattern Algorithm seeking	"15, I add to 6, the answer is 21. 21 added to 7 equals 28. So the 7th triangular number has 28 circles."	Think-aloud
	Describe pattern	"Add 2, then 3, and then 4."	Interview
Rina	Look for pattern	"The difference between Figure 3 and Figure 2 is that Figure 3 has 3 more circles."	Think-aloud
	Recognise pattern	"Figure 4 has 4 more circles. Figure 5 must have 5 circles more."	Think-aloud
	Extend pattern	"So I will draw on the top 1 circle, below it has 2 circles, then below the 2 circles has 3 circles, and next 4 circles and next 5 circles.	Think-aloud

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Table 3. *(Continued)*

Participant	Pre-algebraic thinking inferred	Verbal protocols (translated from the Malay language)	Source of verbal protocols
	Extend pattern	"The answer is on top has 1 circle, then 2 circles, then 3 circles, then 4 circles, 5 circles, 6 circles and 7 circles."	Think-aloud
Firul	Extend pattern	"3…3, 4, 5, 6…6, 7, 8, 9, 10 so 15."	Think-aloud
	Describe pattern	"Because 1 to 3 got 1. 1, 2, then 3. So I do not need to count 3. 3 to 6…3, 4, 5…got 2. 6 to 10…6, 7, 8, 9, got 3. So 10, 11, 12, 13, 14, so get 15…the 5th triangular number."	Think-aloud
	Extend pattern	"15, 16, 17, 18, 19, 20, 21…6. Oh, 7. 21, 22, 23, 24, 25, 26, 27, 28…the 7th triangular number."	Think-aloud

Ali, Zita and Rina looked for a pattern via the difference between consecutive triangular numbers (for the cases of Ali and Zita) or diagrams representing the triangular numbers (for the case of Rina). This led Ali and Zita to describe the pattern in terms of increasing differences between the consecutive triangular numbers, whereas Rina recognised the pattern based on the increasing number of circles among consecutive diagrams. Hence, they extended the pattern differently. Ali and Zita extended the pattern using addition, thus leading to the inference of 'algorithm seeking'. Rina extended the pattern by explaining how the diagrams representing the 5th and 7th triangular numbers should look.

Firul seemed to be able to extend the pattern immediately to obtain the fifth triangular number. He seemed to see the pattern differently than his counterparts, judging from his way of describing the pattern. Hence, he extended the pattern via the 'counting on' strategy.

Another interesting result related to this problem is the use of multiple representations by Zita in her solution process. She not only extended the pattern in an arithmetic way, but also drew diagrams representing the fifth and seventh triangular numbers to verify her answers, as reflected in the following interview extract:

Researcher:	Ok. After you got the answer 15, you drew a diagram. Can you tell me why you did so?
Zita:	To confirm the triangular number.

Further interviewing revealed that she also could describe the pattern in the triangular numbers by explaining the increasing difference in the triangular numbers (Table 3).

The 'Tile" Problem (Adapted from Kaput & Blanton, 2001)

Look at the pattern below.

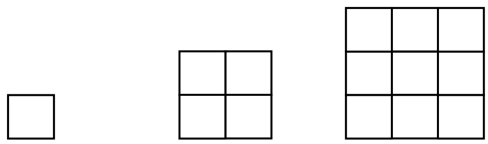

Figure 1 Figure 2 Figure 3

How many of the smallest squares will be in Figure 5 if this pattern continues? Show and explain how you find your answer.

This problem also involves an irregular geometric pattern, as the pattern grows in an increasing but generalisable way. However, the participants are required to make a 'jump' in their solution process, as this problem does not involve the preceding figure, as in the two previous problems. For this problem, Zita did not arrive at the correct answer due to a careless mistake in counting while drawing her Figure 5. Consequently, only four participants' verbal protocols were analysed (Table 4).

Table 4. Verbal protocol analysis for the 'Tile' problem

Participant	Pre algebraic thinking inferred	Verbal protocols (translated from the Malay language)	Source of verbal protocols
Ali	Look for pattern	"I try to find the distance between Figure 1 and Figure 2. Figure 1 has 1 tile (Pause) and Figure 2 has 4 tiles. (Pause) Distance between these two figures is…3. Figure 3 has 9 tiles. (Pause) Distance between Figure 2 and Figure 3 is…5."	Think-aloud

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Table 4. *(Continued)*

Participant	Pre-algebraic thinking inferred	Verbal protocols (translated from the Malay language)	Source of verbal protocols
	Recognise pattern	"So the distance between the 3 figures is 3 and 5."	Think-aloud
	Describe pattern	"So we try to find Figure 4. 3 added by 2 is 5."	Think-aloud
	Extend pattern	"Figure 4…distance must be added by 2. So the distance is…7. Figure 4 has 15…4, 8, 12, 16 tiles."	Think-aloud
	Describe pattern	"I want to find Figure 5. The distance is 3, 5, 7. The pattern is added by 2. So 7 added by 2, the distance is 9."	Think-aloud
	Extend pattern	"This is Figure 5. (Pause) 1, 2, 3, 4, 5. Total tiles…1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25. So Figure 5 has 25 tiles."	Think-aloud
Rina	Look for pattern	"Em…what's the difference between Figure 3 and Figure 2?"	Think-aloud
	Recognise pattern	"Figure 3 has 5 tiles more. Figure 2 has 5 tiles less."	Think-aloud
	Extend pattern	"Em…what if I add…em how many…3 tiles on top and…4 tiles below. This is Figure 4. If for Figure 5, I will copy Figure 4. I will add 4 tiles on top and 1, 2, 3, 4, 5 tiles at the side. This is Figure 5. For Figure 5, it has 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25…25 tiles."	Think-aloud
	Describe pattern	"Because I see Figure 2 is like 2 times 2, get 4. After that 3 times 3 get 9, and if Figure 4 must be 4 times 4. If Figure 5 must be 5 times 5."	Interview

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Table 4. *(Continued)*

Participant	Pre-algebraic thinking inferred	Verbal protocols (translated from the Malay language)	Source of verbal protocols
Firul	Recognise pattern	"1 times 1 get 1. 2 times 2 get 4. 3 times 3 get 9."	Think-aloud
	Describe pattern	"So they multiply by themselves."	Think-aloud
	Extend pattern Algorithm seeking	"This is my answer for Figure 5…25 tiles."	Think-aloud
Diana	Recognise pattern	"Figure 1 has 1 row and 1 tile, and in Figure 2 has 2 tiles, Figure 3 has 3 tiles in a straight line."	Think-aloud
	Extend pattern	"So in Figure 4 has (Pause) 4 tiles at the side. Figure 4 has 16 tiles while Figure 5…Figure 5 has 5 tiles in a straight line and (Pause) in Figure 5 has 25 tiles."	Think-aloud

Both Ali and Rina looked for a pattern via the difference in the number of tiles in the given figures, leading to the way they described the pattern in terms of the increase in the number of tiles from one figure to the next. In extending the pattern, Ali seemed to make use of the differences in the number of tiles from one figure to the next and described the pattern as "added by 2". Rina seemed to extend the pattern by adding additional tiles to the top and bottom or side of the previous figure. The interview with her seemed to confirm her strategy of extending the pattern; she described the pattern based on the squares of numbers.

Firul seemed to recognise the pattern in the way Rina had described it. That enabled him to describe the pattern and hence extend it quite easily. The retrospective interview suggested that he got the answer via "5 × 5 = 25", as shown in the following interview extract:

> Researcher: Can you explain why 25 is the answer?
> Firul: Because Figure 5 multiplies by itself. 5 times 5 is 25.

Diana described the pattern in another way. She focused on the number of tiles in a row – as reflected by her verbal protocols "3 tiles in a straight line", whose meaning is clarified in this interview extract:

Researcher: You said "Figure 3 has 3 tiles in a straight line". Can you explain what you mean?
Diana: It means 3 tiles as the width and 3 tiles as the length.

Based on her own interpretation of the given figures, she could extend the pattern to get Figure 4 and then Figure 5.

Summary of Results

Results of the verbal protocol analysis for the three problems are summarised in Table 5. The name of the participant from whom the particular pre-algebraic thinking was inferred is also shown.

'Look for pattern', 'recognise pattern' and 'extend pattern' constituted the most commonly inferred pre-algebraic thinking (Table 5). However, some participants skipped the process of looking for patterns if they could recognise the pattern (for example, Zita and Rina in solving the 'Table' Problem; Firul and Diana in solving the 'Tile' Problem). Before they extended a pattern, the participants seemed to go through the processes of either looking for, recognising or describing the pattern. 'Generalise pattern' was not inferred in any of the participants' verbal protocols.

Table 5. Summary of results

Pre-algebraic thinking inferred	The 'Table' Problem	The 'Triangular Number' Problem	The 'Tile' Problem
Look for pattern	Ali, Firul & Diana	Ali, Zita & Rina	Ali & Rina
Recognise pattern	Zita, Rina & Diana	Zita & Rina	Ali, Rina, Firul & Diana
Describe pattern	–	Ali, Zita, Rina & Firul	Ali, Rina & Firul
Extend pattern	Ali, Zita, Rina, Firul & Diana	Ali, Zita, Rina & Firul	Ali, Rina, Firul & Diana
Generalise pattern	–	–	–
Algorithm seeking	Firul	Ali & Zita	Firul
Test conjecture	Diana	–	–
Justify generalisation	Diana	–	–
Use multiple representations	–	Zita	–

There were instances in which a particular type of pre-algebraic thinking was inferred for only one participant. For example, Diana appeared to be the sole participant who tested her conjecture and justified her generalisation in the 'Table' Problem. Firul was the sole participant who used operations, hence the inference of 'algorithm seeking' in solving the 'Table' Problem and the 'Tile' Problem. Zita was the only participant who used both standard algorithms and diagrams in her solutions for the 'Triangular Number' Problem, hence leading to the inference of 'use multiple representations'. Figure 1 shows Zita's written solutions for the 'Triangular Number' problem.

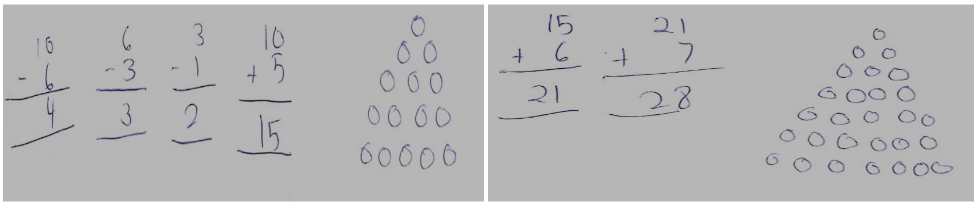

Figure 1. Zita's written solutions for the 'Triangular Number' problem

Some of the participants seemed able to integrate arithmetic algorithms when dealing with geometric patterns. For example, Firul used addition and multiplication to reach his solutions in the "Table' Problem and the 'Tile' Problem, respectively, whereas Ali and Zita used addition to achieve their solutions in the 'Triangular Number' Problem.

DISCUSSION

Results of analysis indicated that most of the participants could look for, recognise, describe and extend patterns to solve generalisation problems involving geometrical patterns. These abilities seemed to indicate their emerging algebraic thinking (Blanton & Kaput, 2003; CSBE, 2000). Additionally, ability to look for, recognise, describe and extend patterns among these participants reflected their abilities to detect sameness and differences, as well as to make distinctions (Mason, 1986).

Pre-algebraic thinking, such as algorithm seeking, testing conjectures, justifying generalisations, and using multiple representations, were inferred only from particular participants and only on particular problems. This implies that the participants' pre-algebraic thinking may be further enhanced through relevant teaching and learning activities in primary mathematics classrooms. In particular, algorithm seeking is closely related to pupils' numeric and arithmetic understandings (Slavit, 1999) such as understanding the meanings and properties

of the four basic arithmetic operations and their relationships. Other than that, generalisation activities such as making general statements about shapes or various number patterns can lead to formation followed by testing of conjectures. This may give rise to the opportunity to explain and justify one's idea. In the process of justifying, the pupil may transform his/her explanation in words into a diagram, which involves using another representation to express his/her idea.
One notable finding in this study was that 'generalise pattern' was not inferred from any participant. This implies that these participants had yet to use their own words to make a general statement regarding the geometric patterns identified. This seems to be related to Warren's (2000b) point of view that limited ability to reason visually could be a consequence of the lack of early algebraic experiences, which in turn seems to reflect the lack of emphasis on early or pre-algebra in the Malaysian primary school Mathematics curriculum.

CONCLUSION

Due to the limited number of pre-algebraic problems and participants presented in this paper, the results presented and discussed in this paper may be more indicative than they are definitive. However, from the results presented in this paper, the participants have shown their emerging abilities to look for and recognise geometric patterns. It is also noted that these participants' pre-algebraic thinking did not encompass all pre-algebraic thinking as presented in Table 1, and that all the participants have yet to exhibit their abilities in generalizing geometric patterns.

Based on the results presented in this paper, we advocate that Malaysian primary school children be given more opportunities to explore and work with either geometric or number patterns so that they are able to not only look for, recognize and extend patterns, but also describe and generalise patterns in their own way. In line with views of Femini-Mundy et al. (1997), Kenney and Silver (1997), Urquhart (2000) and Burns (2002), we recommend patterning activities to be infused into the Malaysian primary school Mathematics curriculum because we believe that the study of patterns helps to lay a foundation in developing primary school pupils' algebraic thinking, which is vital for the learning of more abstract mathematics at higher levels.

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