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AN EXPLORATORY STUDY INTO MALAYSIAN CHILDREN'S UNDERSTANDING OF MULTIPLE REPRESENTATION STRAND OF NUMBER SENSE

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Abstrak: Ramai pendidik matematik di luar negara dan juga di Malaysia yang merasa bimbang sama ada murid-murid sekolah rendah benar-benar memahami matematik yang mereka lakukan ataupun lebih kepada menjalankan algoritma kira mengira yang mana kebanyakan daripada kira mengira tersebut dapat diambil alih dengan menggunakan kalkulator sahaja. Seterusnya, didapati dokumen yang membincangkan perubahan dalam matematik di sekolah di luar negara menekankan keperluan murid memahami kepekaan nombor kerana dianggap kebolehan murid dalam bidang ini membantu mereka memahami matematik pada tahap pemikiran yang lebih tinggi. Penekanan istimewa berkenaan kepekaan nombor pada peringkat rendah adalah penting kerana pengalaman murid-murid mempelajari konsep nombor pada peringkat ini membantu membentuk kepercayaan dan nilai yang positif atau sebaliknya terhadap konsep nombor. Oleh itu, pada peringkat awal ini, murid-murid perlu melihat dan mempercayai bahawa mempelajari matematik sebagai suatu aktiviti yang memberi makna dan mencabar berbanding melihat aktiviti matematik sebagai menghafal algoritma dan aktiviti latih tubi sahaja. Kertas kerja ini akan membincangkan sebahagian daripada penyelidikan tentang kepekaan nombor murid dengan memfokus kepada aspek perwakilan pelbagai dalam kepekaan nombor. Data dikutip secara kuantitatif melibatkan 406 orang murid. Seterusnya, enam orang murid dipilih untuk ditemu bual. Tiga aspek perwakilan pelbagai yang dikaji adalah pecahan sebagai rajah berlorek, nombor perpuluhan di atas garis, perwakilan pecahan dan nombor perpuluhan setara di atas garis. Analisis dapatan kajian ini menunjukkan murid menghadapi masalah memahami nombor di atas garis untuk nombor perpuluhan dan pecahan, dan juga perwakilan pecahan dan perpuluhan yang setara. Walau bagaimanapun, murid menunjukkan kefahaman tentang pecahan sebagai rajah berlorek. Data daripada kajian ini menunjukkan walaupun murid mempunyai strategi menyelesaikan masalah yang dikemukakan, kepekaan nombor mereka membantu membentuk strategi bermakna.

Abstract: Many mathematics educators worldwide as well as in Malaysia are concerned whether primary school students demonstrate understanding of numbers or were they just applying standard algorithm which could have easily been computed with calculator. Moreover, curricular reform documents in other countries emphasize the importance of number sense based on the rationale that numbers sense will be very helpful to understand mathematics and develop higher order thinking. Relatively, the focus on the term "number sense" in the mathematics curriculum is quite recent and most has targeted their arguments to the primary school level. Special emphasis is placed on number sense at the primary schools level for children's experiences related to the learning of number concepts at this level is of crucial importance in instilling their beliefs and values they associate with mathematics. If these experiences are meaningful, it will further lead to positive attitudes, values and beliefs about number concepts. On the contrary, experiences that are not mathematically meaningful will lead them to believe that mathematics learning only consists of memorizing activities devoid of meaning. Therefore it is important at this early stage for children to see and believe that mathematics is a meaningful and challenging activity rather than seeing mathematics as a series of algorithms, drills and practice. This paper will discuss part of a bigger study that aims to investigate students' number sense focusing on it's multiple representation aspect. The quantitative research sample consisted of 406 students. Six students were selected for further interviews. Three aspects of multiple representations investigated were fractions as shaded regions, decimals on a number line, and representation of fractions and decimals on a number line. Findings from this study showed that students face difficulty to understand fractions and decimals on a number line, as well as the representation of equivalent fractions and decimals. However, the students showed understanding of fractions as a shaded region. Data from this study showed that even when the students could have their strategies to solve problems, their number sense actually help them to form a meaningful strategy.

INTRODUCTION

Many studies have shown that students' experiences related to the learning of number concepts at the primary school level are of crucial importance in instilling their beliefs and values they associate with mathematics. If these experiences are meaningful they will further lead to positive attitudes, values and beliefs about number concepts. On the contrary, experiences that are not mathematically meaningful will lead students to believe that mathematics learning consists of memorizing activities devoid of meaning [National Council of Teachers of Mathematics (NCTM) 1989].

However, mathematics educators are concerned that many students demonstrate little understanding of numerical situations in where they have to solve number problems (Leutzinger & Bertheau 1989; Burns 1989; Munirah 2000). Yang (1995) suggested that this could be due primarily to the mindless application of the standard written algorithms which students learned in school. Students are good rule followers but unfortunately do not always understand the procedures they learned (Hiebert 1986). They are adept at manipulating and following symbol rule but are less able at making sense of numerical situations. Moreover, while emphasis on computational skills may produce high computational scores, the extent to which these processes transfer to the students' understanding is unknown. A number of mathematics educators seem to agree that the difficulties experienced by students in solving mathematics exercises is closely related to the development of number sense thinking (Leutzinger & Bertheau 1989; Burns 1989).

WHAT IS NUMBER SENSE?

In multiplying 4.5 × 1.2, a student carefully lined up the decimals and then multiplied, obtaining the answer 54.0 (Reys et al. 1991: 3). When children are asked why they say 17 is larger than 13, they respond that "it just is". They are unable, when asked, to give any further justification (Sowder & Wheeler 1989). Research by Behr, Wachsmuth, Post and Lesh (1984), and Kerslake (1986) showed that, there are children who believe that the denominators and numerators of a fraction are two separate entities, therefore, 6/8 is said to be bigger than 6/7. Student know the answer to 6 × 6 but cannot multiply 7 × 6. There are many examples of such errors, which are said to reflect a lack of "number sense". Responses to questions such as these reveal the level of understanding of number meanings, operations and computations.

Number sense is difficult to define (Hope 1989; Sowder & Kelin 1993; Greeno 1991; Case 1998) and may mean different things in different context but a situation where a person display number sense could be identified (Greeno 1991). Moreover, situations where students display lack of number sense could also be identified (Hope 1989). Apart from being difficult to define, number sense is difficult to measure too (Sowder & Kelin 1993). Nonetheless, despite being difficult to define and measure, number sense is an important trait for students to possess (Hope 1989). Greeno (1991) suggest that number sense as a cognitive skills as a product of learning and not as an objective of teaching. Thus, number sense is seen as having an intuition about numbers (Howden 1989).

Curricular reform documents (such as NCTM 1989; Cockcroft 1982) emphasize the importance of number sense based on the rational that number sense will be very helpful to understand numbers in general. A number of mathematics educators seemed to agree that the difficulties experienced by children in solving mathematics exercises are closely related to the development of number sense thinking (Leutzinger & Bertheau 1989; Burns 1989). Although considerable attention to number sense is occurring in countries like the United States, Australia and the United Kingdom, the term "number sense" is rarely heard in mathematics education, national mathematics curriculum, school classrooms and even teachers or educational journals in Malaysia. Even though many good teachers are undoubtedly teaching mathematics in ways that lead their students to develop good understanding in numbers and operations, the relationships between numbers and operations, and computations, the researcher believes that the development of number sense will play an important role in elementary mathematics education in Malaysia.

From the discussion on the various definitions of number sense, as a consensus, it is agreed that it is difficult to define number sense. However, all the definitions discussed are consistent that number sense is important for students to understand the numbers and operations that they deal with. Among the early efforts to define a comprehensive definition is by NCTM (1989). NCTM (1989) lists five indicators of number sense. These include well understood number meanings, existence of and reliance on multiple numerical relationships, recognition of relative magnitude of numbers, awareness of the relative effect of operating on numbers, and use of referents for measures of common objects and situations in their environments.

NCTM (1989), however, did not discuss an instrument to measure number sense. The development of an instrument to measure number sense is still in the early stages (Yang 1995). McIntosh, Reys, Reys, Bana and Farrell (1997), proposed a framework based on the review of literature on number sense. The number sense framework include indicators of number sense as discussed by NCTM (1989). The framework formulated the following six number sense strands:

1. Understanding and use of the meaning and size of numbers
2. Understanding and use of equivalent forms and representations of numbers
3. Understanding the meaning and effect of operations
4. Understanding and use of equivalent expressions
5. Computing and counting strategies
6. Measurement benchmarks

The definition of number sense in the above framework will be taken as the definition of number sense in this research. Findings for the other components have been discussed elsewhere while the purpose of this paper is to focus on the findings from the multiple representation strand.

MULTIPLE REPRESENTATIONS OF NUMBERS IN THE NUMBER SENSE FRAMEWORK

Multiple representations refers to the recognition that numbers take many different numerical and representational forms (e.g. fractions as decimals, a whole number in expanded form, or a fraction on a number line) and can be thought about and manipulated in many ways to benefit a particular purpose. A representation refers to a mental structure consisting of the tools used for representing mathematical ideas such as tables, graphs and equations (Confrey & Smith 1991).

Janvier, Bednarz and Belanger (1987) have classified the term representation in two major categories: internal representations and external representations. Internal representations deals with "more particularly mental images corresponding to internal formulations we construct of reality". External representations deals with "all external symbolic organizations", illustrated frequently in the forms of symbols, schema and diagrams. Lesh, Post and Behr (1987) suggested that in mathematics teaching and problem solving, five types of external systems of representations are used: texts, concrete representations/models, icons or diagrams, languages and written symbols. These external representations are associated with internal representations (Lesh et al. 1987; Janvier 1987). The NCTM (2000) stated that representation refers to both the process and the product to the act of capturing a mathematical concept or relationship in some form and to the form itself.

In the last two decades, several researchers have addressed the critical problem of translation between and within representations, and emphasized the importance of moving among multiple representations and connecting them (Goldin 1998). Researchers have also found that the translations among representations are important for students' learning (Lesh et al. 1987), since each representation yields its own insights into mathematical concepts (Confrey & Smith 1991). Yerushalmy (1997) showed that most students do not take into consideration the movement from one type of representation to another and thus are unable to generalize the concept. In some cases, students identify a mathematical concept with its representations but do not seem to abstract the concept from them (Vinner 1992).

PURPOSE OF THE STUDY

The research reported here is part of a bigger study that aims to look at children's notion about number sense. The purpose of this paper is to explore children's understanding and use of equivalent forms and representations of numbers (multiple representations) strand of number sense. Four aspects of multiple representations that were explored were fractions as shaded regions, number density on a number line for decimals, representations of equivalent fractions and decimals on a number line, and multiple representations of numbers.

METHODOLOGY AND RESEARCH INSTRUMENT

This research employed both quantitative as well as qualitative methods. Firstly, the quantitative data were obtained from the number sense test. Questions that arise from the analysis of the quantitative data were further explored in an interview. The number sense test consisting of 47 items test (adapted from McIntosh et al. 1997) were given to the children to solve. This paper will discuss the findings from the multiple representation strand only. A total of 406 students from three schools in Pulau Pinang and one school in Kedah took part in this study. Six students who were categorized as having excellent, good and average scores respectively in the number sense test were chosen for further interview to further explore into their understanding of multiple representations of numbers.

DISCUSSION OF THE FINDINGS: UNDERSTANDING OF MULTIPLE REPRESENTATIONS IN A NUMBER SENSE TEST

The items in the number sense test were given 1 for a correct score and 0 for an incomplete or incorrect answer. The averages for the five strands differ with multiple representation as the most difficult strand. The percentage of correct answers for the children's understanding and use of equivalent forms and representations of numbers (multiple representations) are indicated in Table 1.

Table 1. Percentage of correct answers for the children's understanding and use of equivalent forms and representations of numbers (multiple representations)

Children's understanding and use of equivalent forms and representations of numbers (multiple representations)	Percentage of correct answers
(Question 9, Question 15 and Question 17) Fractions as shaded regions and fraction as part of a collection of objects	34%
Number density on a number line (decimals)	23.3% (Question 12: 4%) (Question 13: 54%) (Question 14: 22%)
Representing equivalent fractions and decimals on a number line	Question 10 (decimals): 45%
Multiple representations of numbers (fractions or decimals)	Question 11 (fractions): 23%

There were nine questions (Questions 9 to 17) that test students' understanding of multiple representations. The percentage of correct answers for the three questions (Question 9, Question 15 and Question 17) that test students understanding of fractions as a shaded region and fraction as part of a collection of objects is 34%.

Questions 12, 13 and 14 tests students' understanding of decimals on a number line. Even though the percentage of correct answers for all the three questions is 23.3%, there is a wide difference on the students' percentage of correct answer depending on the magnitude of the decimals.

Questions 13 and 14 explore the students' ability in representing number density on a number line for decimals. The findings for questions 13 and 14 are shown in Table 2.

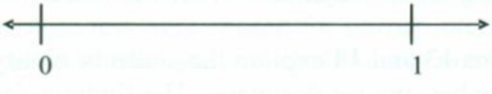
Table 2. Percentage of correct answers for number density on a number line for decimals

Questions	Percentage of correct answers
Question 13. Estimate the decimal shown by the arrows on the number line  	54%
Question 14. Estimate the decimal shown by the arrows on the number line  	22%

Students' performance for question 13 is very good, on which 54% answered correctly. The researcher's opinion is that this may be due to the fact that question 13 required the students to name a decimal that represented a midpoint of the number line 0 to 1. However, when students were asked to name a decimal that represent the midpoint of the number line 0–0.1, the percentage of correct answer dropped to 22%.

Questions 10 and 11 explores the students ability in representing equivalent fractions and decimals on a number line. The findings for questions 10 and 11 are shown in Table 3.

Table 3. Percentage of correct answers for representing equivalent fractions and decimals on a number line

Questions	Percentage of correct answers
Question 10. Place the numbers 0.1 and 0.8 in their correct positions on this number line	 45%
Question 11. Place the numbers 1/10 and 4/5 in their correct positions on this number line	 23%

The comparison for the students' performance in questions 10 and 11 is shown in Table 4.

Table 4. Comparison for the students performance in questions 10 and 11

Questions	Question 10		Question 11	
	Question 10a (0.1)	Question 10b (0.8)	Question 11a (1/10)	Question 11b (4/5)
Percentage correct	48.5	44.8	30.5	22.7

The percentage of correct answers for questions 10a and 10b are 48.5% and 44.8%, respectively. The percentage of correct answers dropped to 30.5% and 22.7%, respectively when the students were asked to place the fractions 1/10 and 4/5 on the given number line. A cross-tabulation for the results of questions 10 and 11 was carried out to further explore the actual percentage of students that were able to answer both questions correctly as shown in Table 5.

Table 5. Cross-tabulation for question 10 (decimal 0.1) and question 11 (fraction 1/10)

		Question 11 (fraction 1/10) %		Total
		Wrong	Correct	
Question 10 (decimal 0.1)	Wrong	45.5	5.7	51.2
	Correct	23.5	25.0	48.8
		69.0	30.7	100

Table 5 shows that 45.5% of the students were not able to place both numbers 0.1 and 1/10 on the number line. 25% students were able to place both numbers 0.1 and 1/10 on the number line. 5.7% students were not able to place the number 0.1

but were able to place the number 1/10 on the number line. 23.5% students were able to place the number 0.1 but were not able to place the number 1/10 on the number line. One question that arises is whether the students do not know that 0.1 is equal to 1/10 or they were not able to place the fraction 1/10 on the number line. This question will be explored later in the interview.

DISCUSSION OF THE FINDINGS FROM THE INTERVIEW

The six students were interviewed on a one-to-one basis using the structured interview method. Specifically, the interview sessions sought to explore students' mental representations of fractions and the students' solution strategies for the given questions. The interview items focused on exploring students understanding and use of equivalent forms together with representations of numbers (multiple representations) through exploring the following concepts:

Task 1. Students' mental representation of fractions.
Task 2. Fractions as decimals.

Data Analysis

The interview sessions with the students were videotaped and transcribed. The interview protocol for each student was constructed according to the tasks given. The analysis for each task for the students was carried out to find some common as well as specific strategies that the students could use in performing the given tasks. The strategies employed by the students in completing the tasks were categorized as strategies that reflected their understanding of fractions and strategies that lack of understanding of fractions.

FINDINGS

Task 1. Students' mental representations of fractions

There were three different mental representations of fractions. Two of the representations were categorized as representations that reflect number sense as shown below:

Task 1 (a): Students mental representations of fractions that reflect number sense in the representation of the fraction $\frac{2}{4}$

Student	Representation of the fraction $\frac{2}{4}$
Fai Jay Syaz Wei	
Saf	Four marbles that are divided into two groups

Task 1(b): Students' mental representations of fractions that lack number sense in the representation of the fraction $\frac{2}{4}$

Student	Representation of the fraction $\frac{2}{4}$
Lil	The numbers that represent the given fraction are given that is stud sees $\frac{2}{4}$ as the numbers 2 and 4

Task 2. Fractions as decimals

All of the students who took part in this study had not studied how to convert a fraction with its denominator not equal to 10 to its equivalent decimals. All of the students had a strategy to convert the given fraction into a decimal. The strategies employed by the students to convert a fraction into its' decimal equivalent were categorized as strategies that reflect the number and strategies that lacks number sense. The students' strategies for converting a fraction to its decimal equivalent that reflect number sense are given below.

Task 2(a): Conversion of fraction to decimals that reflect number sense

Student	Fraction $\frac{a}{10}$ as decimals	Fraction $\frac{4}{5}$ as decimals
Fai	Converts $\frac{a}{10}$ to 0.a	Converts $\frac{4}{5}$ to 4.5 but knows $\frac{4}{5} = \frac{8}{10}$, and therefore says $\frac{4}{5} = 0.8$ but still thinks $\frac{a}{b}$ = a.b
Syaz	Converts $\frac{a}{10}$ to 0.a	Converts $\frac{4}{5}$ to 0.8 by dividing 4 by 5
Wei	Converts $\frac{a}{10}$ to 0.a	Does not know how to convert $\frac{4}{5}$ to its decimal form but knows that $\frac{4}{5} < \frac{5}{5}$, therefore, $\frac{4}{5} < 1$.

The students' strategies for converting a fraction to its decimal equivalent that reflect a lack of number sense is given in task 2(b).

Task 2(b): Conversion of fractions to decimals that lack number sense

Student	Fraction $\frac{a}{10}$ as decimals	Fraction $\frac{4}{5}$ as decimals
Jay	Converts $\frac{a}{10}$ to 0.a	Converts $\frac{4}{5}$ to 4.5
Lil	Converts $\frac{a}{10}$ to 0.a	Converts $\frac{4}{5}$ to 4.5
Saf	Converts $\frac{a}{10}$ to 0.a	Converts $\frac{4}{5}$ to 4.5

The strategy "comparing numerator, if a > b, then $\frac{a}{4} > \frac{b}{4}$" was categorized as a strategy that reflected an understanding of a fraction for Wei because he also used the strategy "model of a square that is divided into four equal parts with two shaded parts representing $\frac{2}{4}$ and three shaded parts representing $\frac{3}{4}$". The strategy "for the fraction $\frac{a}{b}$, b marbles divided into a group" refers to representing fractions as part of a set. This strategy was categorized as a strategy that lacks an understanding of fractions in this study because the student who employed this strategy (Saf) used another strategy that again did not reflect an understanding of fractions in comparing two fractions. Saf employed the strategy "fraction $\frac{a}{b} < \frac{c}{d}$ if b divides a, b does not divide c". One of the findings of this research was that students face difficulties to form appropriate mental representations of fractions and as a result face difficulties in comparing the size of two fractions.

CONCLUSION

This study tried to investigate students' multiple representations of numbers from the perspective of the number sense framework. The four aspects of representations that were explored were fractions as shaded regions, number density on a number line for decimals, representations of equivalent fractions and decimals on a number line, and multiple representations of numbers.

An analysis of the students' performances in the number sense test showed that many students have difficulties in understanding the concepts of number density on a number line for decimal numbers, and representing equivalent fractions and decimals on a number line. However, the students in this study showed a good understanding of representing fractions as shaded regions. Data from this study showed that while students are adept at creating their own strategies, their number sense ability was essential for the strategies adapted to be meaningful or otherwise. Data from this study identified the qualitative differences in the strategies employed by the students.

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